ORIGINAL ARTICLE

"TO AND FRO" BLOOD FLOW MATHEMATICAL MODEL IN VENOUS FLAP SURVIVAL

DOI: 10.36740/WLek202108131

Oleksander A. Radomski[1], Nataliia Yu. Radomska[2], Iryna V. Kerechanyn[2], Lilia M. Yaremenko[3], Larysa Yu. Sankova[2], Volodymyr M. Kovalchuk[2]
[1]SHUPYK NATIONAL HEALTHCARE UNIVERSITY OF UKRAINE, KYIV, UKRAINE
[2]PRIVET HIGHER EDUCATIONAL ESTABLISHMENT «KYIV MEDICAL UNIVERSITY», KYIV, UKRAINE
[3]O.BOGOMOLETS NATIONAL MEDICAL UNIVERSITY, KYIV, UKRAINE

ABSTRACT
The aim of the research is to determine "to and fro" venous flap blood circulation perfusion ability and flap size survival.
Materials and methods: "To and fro" blood circulation mathematical model based on the theory of compressed liquid flow thought the porous medium.
Result: Designed mathematical model allows to calculate the viable flap size that depends on pedicle vessel radius, the blood pressure gradient, the blood viscosity, the elastic tissue capacity, the pulse frequency, vessels flap topography and surgical pedicle flap technique. Viable flap width may extent up to 4-6 cm.
Conclusions: Only thin skin or fascial flaps, were through venous pedicle with a lot of side branches located under the flap surface and along central flap axis may survive with "to and fro" blood circulation.

KEY WORDS: venous flap, blood circulation, mathematical model

Wiad Lek. 2021;74(8):1960-1963

INTRODUCTION
Arterialized venous free flap transplantation is popular in the area of plastic surgery for covering the skin defects [1-5]. However, mechanisms of the survival of the flap is still unclear and the necrosis of the soft tissue after transplantations flap is still possible [6,7].

For the explanation of the tissue survival Thatte M.R. at all (1989) offered the mathematical model of pendulum blood venous supply of the flap [8]. According to this model the physiological blood circulation within flap vessels is absent. Instead it, blood moves through the central venous flap`s pedicle and its terminal brunches according the "to and fro" principle. During systolic high pressure the portion of "fresh" blood passes though the flap vessels. And diastolic low pressure pushes blood out from the flap`s vessels. Authors supposed that flap vessels net can be compered with the ideal mix reactor, where "work off" blood is blended in a moment with "fresh" portion. As the result, according to this model the flap and its pedicle may have any size for survival. But experimental and clinical researches demonstrated about 5% flap`s necrosis still occurs.

THE AIM
The aim of the research was to identify the efficiency of the "to and fro" venous flap`s blood circulation perfusion and flap`s size for survival.

MATERIALS AND METHODS
Mathematical model. Takes to account, that flape with λ thickness has radius R of the vascular padicle (fig.1). Sinusoidal blood pressure flap inflow has maximal height P_m (difference between systolic and diastolic pressure) and duration time T that depends on pulse frequency. The flape tissue compare with entire porous medium that consists of the pore middle radius r and has ε porosity (vascular flap net volume to entire flap volume ratio).

The pressure gradient moves blood along axial vessel with $U_y(y,t)$ speed. Then blood enters into a flap`s tissue from branches side, and reaches speed component along axis X: Ux(x,y,t). The main idea of the task is to determine axial and side vessels branches blood moves with speed in any time. After stated calculations there is a possibility to determine "fresh" blood boundary transfered from initial time moment at flap entrance point (x=0, y=0). Maximal "fresh" blood bound flap tissue transfers was corresponded to viabil flap size.

RESULTS

MATHEMATICAL MODEL FORMULATION
Because of vessels wall elasticity, the flap blood circulation measuring is equivalent to compressed liquid flow task. Compressed liquid flow differential equation in porous

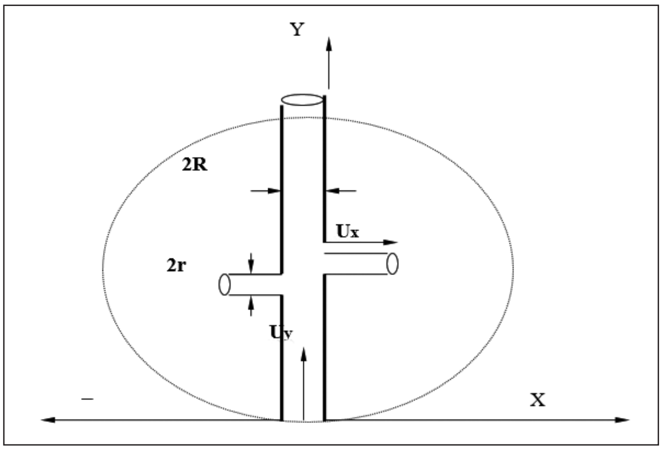

Fig. 1. Schematic appearance blood move along axial pedicle (axis Y) and side branches (axis X).

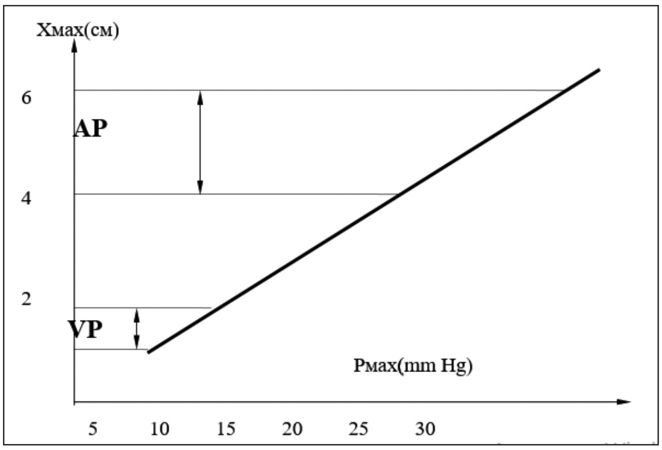

Fig. 2. Maximal vital width flap (Xmax.) gradient pressure (Pmax.) depends. VP – venous pressure flape maximal width, AP – arterial pressure flape maximal width: conditions r=1.0mm; $\mu=2\cdot10^{-3}$H·c/m^2; ß=2.5×104m^2/Hмах; £=5.0mm; T=0.8 c.

medium for one-dimensional case was calculated by the following formula I [9]:

$$\varepsilon\frac{\partial\rho}{\partial t}=-\frac{\partial(\rho u_x)}{\partial x}; \qquad \text{(I)}$$

where ρ – blood density; ε – flap tissue porosity; U_x – blood move speed.

Liquid speed move, according to Puazeil low [10], is proportional to pressure gradient that was calculated by the following formula II.

$$U_x=-\frac{r^2}{8\mu}\frac{\partial P}{\partial x}; \qquad \text{(II)}$$

where μ – tenacity.

Take in account that liquid density in equation I is identical with a liquid quantity in volume flap unity, – the dependence low of this quantity from pressure was calculated by the following formula III:

$$\rho=\rho_0 e^{\beta P}; \qquad \text{(III)}$$

where coefficient β have a value of relative blood tissue volume change from unit pressure change.

After the mathematical transformations receive differential equation for pressure change in dimensionless quantities was calculated by the following formula IV:

$$\frac{\partial P(X,\tau)}{\partial\tau}=b\frac{\partial^2 P(X,\tau)}{\partial X^2}; \qquad \text{(IV)}$$

where dimensionless parameters are:

$$X=\frac{x}{\lambda};\tau=\frac{t}{T};P=\frac{p}{p_m};b=\frac{r^2T}{8\mu\beta\lambda^2}$$

Initial conditions for equation IV τ=0; P(x,0)=0 and boundary condition τ≥0; x=0; P(0,τ)=P(Y), where P(Y) is a solution of equation for blood pressure changes in vessel pedicle. This equation was deduced on the basis that blood part force out from axial vessel to flap side vessels branches. With this, speed blood displace to flape is proportional to pressure gradient axis X along in axis Y every point (with X=0). Finaly, variables pressure change law in central vessel was calculated by the following formula V:

$$\frac{\partial P(Y,\tau)}{\partial\tau}=a\frac{\partial^2 P(Y,\tau)}{\partial Y^2}+a\sigma\frac{\partial P(Y,\tau)}{\partial x}\Big|_{x=0}; \qquad \text{(V)}$$

where $Y=\frac{y}{R}$ – dimensionless coordinate;

$$a=\frac{T}{8\beta\mu};\sigma=\frac{r^2\varepsilon}{\pi R^2}$$

Under condition Y=0; P=1.

Differential equation system IV, V with corresponding limited condition completely describe flap blood move task. Integration of this equations determine speed blood field in flap vessels.

MATHEMATICAL MODEL DECISION

Equations IV, V direct integration very complicated. Some simplify is needed. Ignore wall vessels pedicle pressure oscillations. In this case equation V left part will be equal zero and it decision with corresponding boundary conditions was calculated by the following formula VI [11]:

$$P(Y)=e^{-gY}; \qquad \text{(VI)}$$

where *g* constant is expressed through already known quantities according to the next formula:

$$g^2=\frac{\sigma}{2}\sqrt{\frac{\pi}{6}}$$

As a result, in limits of made simplifications, immediately after blood pulse along vascular pedicle, pressure distributed time independently (t – till end pulse) and described by equation VI.

Use equation VI as boundary condition for equation IV. Then receive a flap pressure change low and calculated by the following formula VII:

$$P(x,\tau)=e^{-gy}\operatorname{erfc}\left(\frac{x}{2\sqrt{b\tau}}\right); \qquad \text{(VII)}$$

where erfc $z=1-\operatorname{erf}z=1-\frac{2}{\sqrt{\pi}}\int_0^z e^{-z^2}dz$ – additional mistakes function.

BLOOD MOVE EQUATION DECISION AND DEDUCE A SURVIVE FLAP SIZE DEFINE FORMULA

Blood move speed along axis X and Y defines according equation II. For dimensionless quantities was calculated by the following formula:

$$U_x = bq\frac{\partial P}{\partial x}; \quad \text{(VIII)}$$

where $q=P_m\beta$.

Take in account VI, vascular pedicle blood transfer equation was calculated by the following formula IX:

$$\frac{dY}{d\tau} = \gamma e^{-gY}; \quad \text{(IX)}$$

where γ=aqg.

As initial condition Y=0 with τ=0, and maximal blood transfers during time t=T, maximal flap length (L_m) in naturel units equation will calculated by the following formula X:

$$L_m = R^2 r\sqrt{\frac{\pi T}{2\mu\beta\lambda^2\varepsilon^2}}\ln\left(1+\frac{P_m\varepsilon\lambda\sqrt{2\beta T}}{8R^2\sqrt{\pi\mu}}\right); \quad \text{(X)}$$

Maximal flap length in very good proximity in the case of flape thickness is not exceed 2 times vascular pedicle diameter, according to formula X, was measured:

$L_m \approx 120R$

Flap width on every vascular pedicle distance ($0<Y<Y_m$) was calculated by integration of equation VIII and have such view calculated by the following formula XI:

$$X(Y) = X_m e^{-gY}\left(1-\sqrt{\frac{e^{gY}-1}{g\gamma}}\right) \quad \text{(XI)}$$

Finally maximal vital flap withd (X_m) is equal to value calculated by the following formula XII:

$$X_m = P_m r\sqrt{\frac{2T\beta}{\pi\mu}} \quad \text{(XII)}$$

According to our calculations, very thin skin ore fascial flaps, were through venous pedicle with a lot of side branches is located under the flap surface and along central flap axis may survive.

As the result, the diagram (fig. 2) reflects the viable flap width blood pressure depends on factors such as: venous pressure gradient (VP) distal extremities parts maximal flap width (Xmax) 1-2cm; arterial pressure gradient (AP) maximal flap width up to 4-6 см [5,6,7].

Terminal venous pedicle flap has only inflow foramen. Such variant pedicle blood flap perfusion has a considerable less capacity. Pendulum blood moves occur in al vascular flap net and for this reason maximal viable flap width is inly at the base, near pedicle entrance. Gradually side vascularization zone narrows and at a distance of 10cm from a flap base became only 10% from initial maximal width. As a variant viable flap size may be 5 × 2cm or 3 × 3cm with arterial pressure gradient 30 mm Hg. Venous pressure gradient is not enough to nourish terminal pedicle flape [4,5].

Another problem of pendulum circulation is a thrombosis possibility in the peripheral vessels were thrombocytes may aggregates because of slow blood speed flow. As the result, it lead to tissue necrosis because of vessels occlusion [8,9].

DISCUSSION

Due to the research, maximal viable venous pedicle flap length X and width XI,XII is possible to define by offered mathematical model. Two main surgical flap technics for transplantation have been take for calculation [2,4].

Through venous pedicle located along flap axis and have inlet and outlet anastomoses. For this reason blood move in axial vessel trunk have a single direction and in side branches "to and fro" circulation. In this case, according to formula XII, maximal survive flap width will be the same along pedicle axis and is directly proportional to pressure gradient and side vessels diameter.

Evidently, that it is impossible to take in account al blood circulation parameters. For this reason every mathematical model is approximate and can't be put into practice. Never the less proposed blood move model based on compressed liquid flow in porous medium have a real experimental and clinical corroboration and confirm the theory of "to and fro" blood circulation flap survival [5,6].

Terminal pedicle based flap have a very small zone of "to and fro" blood circulation. That is why island venous pedicle flap survival possible based on vasa vasorum or perforator branches vessels arterial supply. Father more, free flap transplantation in such case, is impossible because of anastomosis thromb failure.

Another, biological question is about flap tissue metabolism in pendulum circulation. Perhaps, it mostly occurs out of capillary vessels, because of large erythrocyte size, bigger than capillary foramen. Physiological erythrocyte capillary inflow is possible after its transformation, that became longer and narrow inside capillary. If erythrocyte enters, under the pressure gradient is possible, – erythrocyte departures from capillary in pendulum circulation had no pressure support. Step by step, capillary vessels became blocked and capillary metabolism within the neovascularizated flap was restored after some days [7,8].

CONCLUSIONS

The mathematical model developed of "to and fro" venous blood circulation of the pedicle flap allows to calculate the viable flap size that depends on the pedicle vessel radius, blood pressure gradient, blood viscosity, elastic tissue capacity, pulse frequency, vessels flap topography and surgical pedicle flap technique. Multidirectional blood circulation of the flaps provides the survive of the thin fascial flaps with limited size about 4-6 cm.

REFERENCES

1. Kawakatsu M. Free medialis pedis venous flap transfer for reconstruction of volar finger defects: Clinical application and esthetic evaluation. Journal of Plastic, Reconstructive and Aesthetic Surgery. 2019;72(3): 459-466.

2. Troisi L., Berner J.E., West E.V., Wilson P. Medial plantar flap for hand reconstruction: A systematic literature review and its application for post-sarcoma excision. Annals of Plastic Surgery 2019; 82(3): 337-343.
3. Bartlett E.L., Zavlin D., Menn Z.K., Spiegel A.J. Algorithmic Approach for Intraoperative Salvage of Venous Congestion in DIEP Flap. Journal of Reconstructive Microsurgery. 2018; 34(6): 404-412.
4. Woo, S.H., Lee Y.K., Kim J.Y. et al. Palmaris longus tendocutaneous arterialized venous free flap to reconstruct the interphalangeal collateral ligament in composite defects. Journal of Hand Surgery: European Volume. 2018; 43(50): 518-523.
5. Yang J., Tang Y., He H., Fu S. Repair of soft tissue defects of the fingers with medial plantar venous flap. Revista da Associacao Medica Brasileira. 2018; 64(6): 501-508.
6. Lee D.C., Kim J.S., Roh S.Y. et al. Flap Coverage of Dysvascular Digits Including Venous Flow-Through Flaps. Hand Clinics. 2019; 35(2): 185-197.
7. Choi M.S., Roh S.Y., Koh S.H. et al. Arterial or venous free flaps for volar tissue defects of the proximal interphalangeal joint: A comparison of surgical outcomes. Archives of Plastic Surgery. 2020; 47(5): 451-459.
8. Thatte R.L., Kamdar N.B., Khakkar D.V. et al. Static and dynamic computerized radioactive tracer studies, vital dyes staining and theoretical mathematical calculations to ascertain the mode of survival of single cephalad channel venous island flaps. Br. J. Plast. Surg. 1989; 42(3): 405-412.
9. Berd R., Styuart V., Laytorut E. Yavleniya perenosa. M.: Himiya, 1974, 688 p. (in Russian)
10. Frolov Yu.G. Kurs kolloidnoy himii. M.: Himiya. 1982, 720 p. (in Russian)
11. Pontryagin L. S. Obyiknovennyie differentsialnyie uravneniya. M.: Nauka, 1974, 332 p. (in Russian)

The work is a fragment of inter-departmental scientific research work of the Department orthopedy and traumatology №2 of Shupyk National Healthcare University of Ukraine, Kyiv, Ukraine "New algorithms of treatment the complication high energy skeletal trauma" (state registration number 0119UA10157).

ORCID and contributionship:
Oleksander A. Radomski: 0000-0002-3311-3867 [A, B, D]
Nataliia Yu. Radomska: 0000-0002-6225-0991 [B, D]
Iryna V. Kerechanyn: 0000-0002-3262-2037 [A, B, D]
Lilia M. Yaremenko: 0000-0001-7076-467X [C, E]
Larysa Yu. Sankova: 0000-0001-7076-467X [E, F]
Volodymyr M. Kovalchuk: 0000-0003-2363-7365 [F]

Conflict of interest:
The Authors declare no conflict of interest.

CORRESPONDING AUTHOR
Iryna V. Kerechanyn
Kyiv Medical University
2 Boryspilska St., 02000 Kyiv, Ukraine
tel: +380661790156
e-mail: dr.kerechanyn@kmu.edu.ua

Received: 19.04.2021
Accepted: 30.07.2021

A – Work concept and design, **B** – Data collection and analysis, **C** – Responsibility for statistical analysis, **D** – Writing the article, **E** – Critical review, **F** – Final approval of the article